Teacher notes

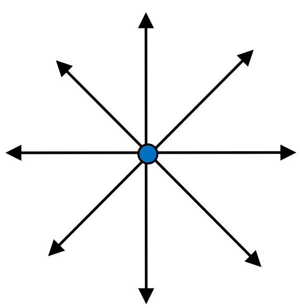
Topic D

Electromagnetism and Newton's third law

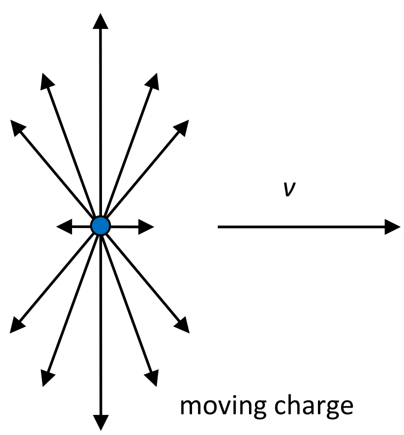
Can Newton's third law be violated?

Feynman in his "*Lectures on Physics*" (Volume 2, Lecture 26) discusses the electric and magnetic fields of a moving charge.

He derives the electric and magnetic fields of a proton moving with speed *v* along the positive x-direction. The detailed formulas are not important for our discussion here. The result is that the electric field is still **radial** as it would have been for a charge at rest, but it is not spherically symmetric. The figure on the left shows the electric field pattern for a charge at rest and the one on the right the pattern for a moving charge. Essentially, the field along the direction of motion is reduced by the relativistic gamma factor $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ and the field at right angles to the direction of motion is increased by the same factor. So, the field is weak along the direction of motion and strong in the orthogonal direction.

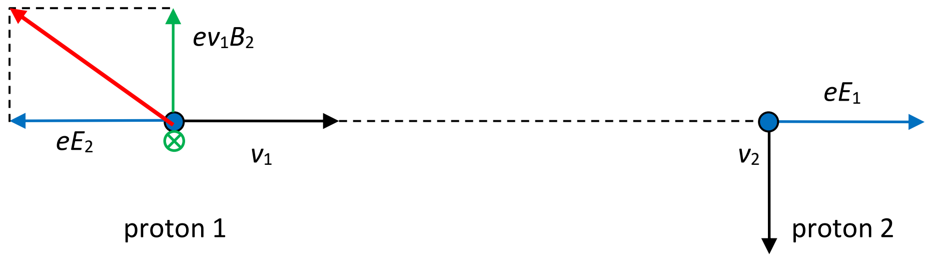


charge at rest



moving charge

The magnetic field is more standard: the moving proton behaves like a current and the magnetic field in this case is that due to a current. Its direction is given by the usual rule that gives the circular magnetic field lines around a wire; the "wire" here being the moving proton. The point now is that there is no magnetic field along the direction of motion of the proton. (Just as there is no magnetic field on the wire, only around it.) Feynman then applies all this to the situation of two protons that move at right angles to each other as shown.



At the instant shown in the figure, proton 2 is on the line along which proton 1 is moving. The protons experience opposite electric forces shown in the blue arrows. Proton 2 creates a magnetic field at the position of proton 1 (directed into the plane of the page) and so there is a magnetic force on proton 1 (green arrow). The **net** force on proton 1 is shown in the red arrow. But as we just saw, proton 1 does not create a magnetic field at the position of proton 2 and so proton 2 **does not** experience a magnetic force.

Action is not equal and opposite to reaction!

Remember that Newton's third law was crucial in deriving the law of conservation of momentum for an isolated system. The two protons here look like an isolated system so the total momentum should be conserved. But it is not because the forces on the two protons are not equal and opposite. The total momentum of the two protons **is** changing.

Feynman gives the resolution to this puzzle in Lecture 27. What we have neglected to consider is that the electromagnetic fields of the two protons carry momentum. The rate of change of this momentum is exactly equal and opposite to the rate of change of the total momentum of the two protons. Thus, the total momentum of the system (protons plus fields) stays constant.

This shows that fields are not just convenient ways to describe forces or help visualize them but are actually real physical quantities.